Identification of Moderately Nonlinear Flight Mechanics Systems with Additive Process and Measurement Noise

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The parameter estimation problem for dynamic systems with both process and measurement noise from nonlinear model postulates is addressed in this paper. A two-step estimation procedure explicitly computes the covariance matrix of residuals and updates the system parameters, the initial conditions, and the state noise matrix using the Gauss-Newton optimization method. For the purpose of state estimation in nonlinear systems with process noise, an approximate steady-state filter is used. In each iteration, the filter-gain matrix is obtained from the postulated system model linearized at the updated initial conditions. The gradients of the output variables and of the system functions are approximated by finite differences. The proposed approach for nonlinear systems with unknown process and measurement noise covariances is first validated on simulated aircraft response data. It is then applied to estimate the aircraft longitudinal derivatives from flight test data using two models with different degrees of nonlinearities. Advantages and possible limitations of the method are discussed.

Nomenclature

A	= state matrix of linearized system
a_x, a_y, a_z	= accelerations along x, y, and z body axes, m/s^2
b_x, b_y	= bias parameters of state and observation equations
C	= observation matrix of linearized system
C_L, C_D	= coefficients of lift and drag
$C_{L_{(.)}}, C_{D_{(.)}}, C_{m_{(.)}}, C_{m_{(.)}}, C_{X_{(.)}}, C_{Z_{(.)}}$	= nondimensional derivatives
C_m	= coefficient of pitching moment
C_X, C_Z	= coefficients of longitudinal and vertical force
$\bar{c}$	= reference chord, m
e^j	= jth unit vector
F	= state noise matrix
F_e	= net thrust, N
$f[.]$	= system state function
G	= measurement noise matrix
g	= acceleration due to gravity, m/s^2
$g[.]$	= system observation function
I_y	= moment of inertia about lateral axis, kg-m^2
J	= cost function
K	= filter gain matrix
k	= discrete time index
$L_{(.)}, M_{(.)}, N_{(.)}$	= dimensional moment derivatives
m	= aircraft mass, kg
m	= number of observation (output) variables
N	= number of data points
n	= number of state variables
P	= covariance matrix of the state error
p, q, r	= roll, pitch, and yaw rates, rad/s
$\dot{p}, \dot{q}, \dot{r}$	= roll, pitch, and yaw accelerations, rad/s^2
$\bar{q}$	= dynamic pressure, N/m^2
R	= covariance matrix of residuals
r_k	= kth diagonal element of R^{-1}
S	= reference area, m^2
t	= time, s
u	= control input vector
u, v, w	= velocity components along x, y, and z body axes, m/s
V	= airspeed, m/s
v	= measurement noise vector
w	= state noise vector
$X_{(.)}, Y_{(.)}, Z_{(.)}$	= dimensional force derivatives
x	= state vector
x_{ck}	= offset distance of cockpit accelerometer from c.g. in x direction
x_α	= offset distance of α sensor from c.g. in x direction
y	= observation vector
z	= measurement vector
α	= angle of attack, rad
β	= vector of unknown system coefficients
Δt	= sampling time, s
Δu	= zero shift in control vector u
Δz	= zero shift in measurement vector z
$\Delta\Theta$	= vector of parameter increments
$\delta_a, \delta_e, \delta_r$	= aileron, elevator, and rudder deflections, rad
δx	= perturbation in x
$\delta\beta$	= perturbation in β
$\delta\Theta$	= perturbation in Θ
Θ	= combined vector of all the unknown parameters
θ	= pitch angle, rad
λ	= vector of unknown elements of F matrix
ρ	= density of air, kg/m^3
σ_T	= tilt angle of the engines, rad
ℓ_{tx}, ℓ_{tz}	= offset distances of engine from c.g. in x and z direction

Subscripts

i, j	= general indices
k	= discrete time index

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m = measured variables
p = perturbed variables
0 = initial conditions

Superscripts

T = transpose
-1 = inverse
$\tilde{}$ = predicted estimates
$\hat{}$ = corrected estimates

Introduction

ESTIMATION of aircraft stability and control parameters from flight test data using different statistical estimation methods has now been rigorously pursued over little more than two decades. The maximum-likelihood (ML) method is one such method that is extensively used for this purpose.[1,2] The ML estimates of system parameters accounting for measurement noise only can be efficiently obtained for linear and bilinear systems with constants coefficients[3-6] and for general nonlinear time invariant systems.[7-9] However, the scope of estimation for dynamic systems with both process (state) and measurement noise has been hitherto mainly restricted to linear systems.[4,5,10-13] Extension of the parameter estimation method to moderately nonlinear systems accounting for both process and measurement noise with unknown covariances is considered in this paper.

Two different approaches to account for process noise in the parameter estimation are possible. They are 1) direct estimation of system parameters and of noise covariance matrices by optimization of a certain cost function, and 2) by formulating the parameter estimation problem as a filtering problem by artificially defining the unknown system parameters as additional state variables. In this paper, the direct approach is investigated as a first step toward estimation of parameters in nonlinear system with process noise. This enables a comparison of the results for a linear example obtained from the approach proposed in this paper with those obtained from the hitherto used formulation for linear systems.[5,13] Further, it also helps to appreciate the difficulties encountered in handling nonlinear systems with process noise.

Following the developments to account for process noise in linear systems,[5] three different formulations, namely, 1) natural formulation,[10] 2) innovation formulation,[11,12] and 3) combined natural cum innovation formulation[5] are possible for nonlinear systems. All of the preceding formulations use the Gauss-Newton method to optimize the likelihood function with respect to the system parameters and/or noise parameters. The main difference among them is in the estimation of unknown noise covariances.

The combined formulation, proposed for linear systems in Ref. 5, takes advantage of the best features of natural formulation (viz., estimation of the state noise matrix using the Gauss-Newton method) and of innovation formulation (viz., explicit estimation of the covariance matrix of residuals). In the case of linear systems, this formulation has been found to be more practical with regard to the convergence, parameter estimates, and computational costs. The Kalman filter with a steady-state (constant) gain matrix, has been used only for the natural purpose of state estimation within the iterative parameter estimation procedure.

With a view to retaining these advantages, the aforementioned combined formulation has been extended in this paper to time-invariant nonlinear systems with additive noise. An approximate but practical filter, used here for the purpose of state estimation in nonlinear systems, is based on the use of a postulated nonlinear system model for the prediction. The correction is, however, based on a constant filter-gain matrix computed using a first-order system approximation about the current initial conditions, which are iteratively updated in the parameter update loop. The Gauss-Newton method is used to optimize the likelihood function. The sensitivity coefficients required in the optimization, and the gradients of the system functions required to compute the filter-gain matrix, are obtained by finite-difference approximation method. This numerical approach enables the handling of general nonlinear system models without additional computer program changes every time the type of nonlinearity in the postulated system model is altered.[7]

The proposed extension of the formulation to estimate parameters in nonlinear systems with both process and measurement noise is initially validated on a linear example using a simulated aircraft response data. Then, its applicability to estimate the parameters pertaining to aircraft lift and drag from flight test data has been demonstrated. For this purpose, two models with different degrees of nonlinearities in the state and control variables, as well as in the parameters, have been used. Advantages and possible limitations of the proposed approach have been brought out.

Maximum-Likelihood Parameter Estimation

The dynamical system, whose parameters are to be estimated, is assumed to be described by the following stochastic equations:

$$\dot{x}(t) = f[x(t),\ u(t),\ \beta] + F\,w(t),\quad x(t_0) = x_0 \tag{1}$$

$$y(t) = g[x(t),\ u(t),\ \beta] \tag{2}$$

$$z(t_k) = y(t_k) + G\,v(t_k), \qquad k = 1,\ldots,N \tag{3}$$

The n and m dimensional system functions f and g are general nonlinear real-valued vector functions. The m dimensional measurement vector z is sampled at N discrete time points with a uniform sampling time of Δt.

It is assumed that the process noise w and the measurement noise v are independent, and that they affect the dynamic system linearly. They are assumed to be characterized by a zero-mean Gaussian white-noise process with an identity spectral density matrix and by a sequence of independent Gaussian random variables with zero-mean and identity covariance, respectively. F and G represent matrices corresponding to the additive process (state) and measurement noise.

It is required to estimate the system parameters β from the discrete measurements of system responses $z(.)$ to given inputs $u(.)$ based on the system model postulated in Eqs. (1-3). In addition to the unknown system parameters, the initial conditions x_0 are also usually unknown. Further, the measurements of variables z and u are likely to contain the systematic errors Δz and Δu, respectively, which may also be required to be simultaneously estimated. If the unknown elements of the F matrix are denoted by λ, then in a general case the complete parameter vector to be estimated is given by

$$\Theta^T = [\beta^T;\ \lambda^T;\ x_0^T;\ \Delta z^T;\ \Delta u^T] \tag{4}$$

In many cases, however, it will not be possible to estimate all of the components of x_0, Δu, and Δz, as they may be linearly dependent or at least highly correlated. In the formulation adapted, the unknown elements of the G matrix are not included in the parameter vector Θ. Instead, the estimate of the G matrix is obtained indirectly from the explicitly estimated covariance matrix of residuals R. This is done to avoid the convergence problems as discussed briefly later in this section.[5]

The ML estimates of Θ and R are obtained by minimization of the negative logarithm of the likelihood function:

$$J(\Theta,R) = \frac{1}{2}\sum_{k=1}^{N}[z(t_k)-y(t_k)]^T R^{-1}[z(t_k)-y(t_k)] + \frac{N}{2}\ln|R| \tag{5}$$

Since it is required to account for process noise, it is essential to incorporate a suitable state estimator (filter). In the case of linear systems, the Kalman filter provides an optimal state estimator. This paper, however, is mainly concerned with the nonlinear models. Truely optimal filters for nonlinear systems, if at all feasible, are of infinite dimensions and, hence, physically unrealizable. In many practical applications approximate filters, usually obtained by linearizing the system equations about some convenient point, are used.[14,15] The three most commonly used first-order approximations are the linearized Kalman filter, extended Kalman filter, and iterated extended Kalman filter.

For the problem at hand, viz., estimation of system parameters by optimization of the likelihood function, irrespective of the choice of a nonlinear filter, the filter is to be used only for the natural purpose of state estimation, and not for the combined state and parameter estimation by artificially defining the unknown system parameters as additional state variables.

In the estimation algorithm accounting for process noise, computation of the covariance matrix of state error and of the filter gain matrix constitutes the major computational burden. Since the assumed model is time invariant, an approximate filter with a constant-gain matrix is used in the present investigations. This also helps to minimize the computational load. This is considered to be a first step toward the complex problem of parameter estimation in nonlinear systems with process noise. In the case of nonlinear systems, the unknown initial conditions x_0 are updated iteratively in the parameter estimation loop. Linearization of the system equations in each iteration about this convenient point, $x_0 = x(t_0)$, can be used to compute the filter-gain matrix. Such an approximate, constant-gain nonlinear filter can be represented in the present case as

$$\tilde{x}(t_k) = \hat{x}(t_k - 1) + \int_{t_{k-1}}^{t_k} f[x(t), u(t), \beta]\, dt \tag{6}$$

$$y(t_k) = g[\tilde{x}(t_k), u(t_k), \beta] \tag{7}$$

$$\hat{x}(t_k) = \tilde{x}(t_k) + K[z(t_k) - y(t_k)] \tag{8}$$

It is to be noted here that the actual postulated nonlinear system equations are used to extrapolate the state variables x by numerical integration and also for computation of y, the predicted system responses. The state variable correction Eq. (8), which depends upon K, is, however, as described next based on the first-order system approximation.

The filter-gain matrix K is a function of the covariance matrix of residuals R, the covariance matrix of state error P, and the observation matrix C of the linearized system. It is given by

$$K = PC^T R^{-1} \tag{9}$$

where

$$C = \left. \frac{\partial g[x(t), u(t), \beta]}{\partial x} \right|_{t=t_0} \tag{10}$$

A steady-state form of the Riccati equation has been used here to obtain the covariance matrix P. The first-order approximation of this equation is obtained as[5]

$$AP + PA^T - \frac{1}{\Delta t} PC^T R^{-1} CP + FF^T = 0 \tag{11}$$

where

$$A = \left. \frac{\partial f[x(t), u(t), \beta]}{\partial x} \right|_{t=t_0} \tag{12}$$

represents the state matrix of the linearized system. The eigenvector decomposition method has been used to solve the Eq. (11), which is in the form of a continuous-time Riccati equation.[5,16,17] The matrix P thus obtained does not depend on time. Equation (9) in turn provides a constant-gain matrix corresponding to the linearization about the updated x_0 in each iteration. The filter algorithm [Eqs. (6-8)] can be used now to compute the model-predicted system variables and, therefrom, the cost function using Eq. (5).

Assuming that the covariance matrix R is known, starting from suitably specified initial values of the parameter vector Θ, the new updated estimates are obtained by using the Gauss-Newton method.[18]

$$\Theta_{i+1} = \Theta_i + \Delta\Theta \tag{13}$$

$$\Delta\Theta = \left\{ \sum_k \left[\frac{\partial y}{\partial \Theta}(t_k) \right]^T R^{-1} \left[\frac{\partial y}{\partial \Theta}(t_k) \right] \right\}^{-1} \times \left\{ \sum_k \left[\frac{\partial y}{\partial \Theta}(t_k) \right]^T \times R^{-1} [z(t_k) - y(t_k)] \right\} \tag{14}$$

Computation of the parameter improvement vector $\Delta\Theta$ thus requires determination of the sensitivity matrix $\partial y / \partial \Theta$. These response gradients and the gradients of the system functions f and g in Eqs. (12) and (10) can be evaluated using either the analytical differentiation or finite-difference approximation. In the case of linear systems, the analytical differentiation method is usually employed to compute the response gradients.[18] This, however, requires explicit solution of the sensitivity equations obtained by partial differentiation of the system equations (1) and (2). Further, also required are the gradients of the filter-gain matrix [Eq. (9)], and of the covariance matrix P [Eq. (11)], which depend on the system functions f and g. For nonlinear systems, the analytical differentiation is tedious and impracticable, requiring system-model-dependent programming changes for each new nonlinear model form. These practical difficulties are eliminated by finite-difference approximation of the gradients.[7]

Since the basic interest of this paper is estimation in nonlinear systems, the finite-difference approach is used to compute all the required gradients. For a small perturbation δx_j in each of the n state variables, the elements of the Jacobian matrices in Eqs. (12) and (10) are approximated using the central-difference formula as

$$A_{ij} \approx \frac{f_i[x + \delta x_j e^j, u, \beta] - f_i[x - \delta x_j e^j, u, \beta]}{2\delta x_j} \tag{15}$$

$$C_{ij} \approx \frac{g_i[x + \delta x_j e^j, u, \beta] - g_i[x - \delta x_j e^j, u, \beta]}{2\delta x_j} \tag{16}$$

Likewise, for a small perturbation $\delta\Theta_j$ in each of the j unknown variables of parameter vector Θ, the perturbed response variable y_{p_i} for each of the unperturbed variables y_i is computed. The corresponding sensitivity coefficient is then approximated by

$$\left[\frac{\partial y}{\partial \Theta}(t_k) \right]_{ij} \approx \frac{y_{p_i}(t_k) - y_i(t_k)}{\delta\Theta_j} \tag{17}$$

The perturbed response variables y_p are obtained from the perturbed system equations, similar to Eqs. (6-8). For each element of β, the state and observation variables can be propagated according to

$$\tilde{x}_p(t_k) = \hat{x}_p(t_{k-1}) + \int_{t_{k-1}}^{t_k} f[x_p(t), u(t), \beta + \delta\beta]\, dt \tag{18}$$

$$y_p(t_k) = g[\tilde{x}_p(t_k), u(t_k), \beta + \delta\beta] \tag{19}$$

$$\hat{x}_p(t_k) = \tilde{x}_p(t_k) + K_p\, [z(t_k) - y_p(t_k)] \tag{20}$$

Computation of the predicted perturbed-state variables $\tilde{x}_p$ from Eq. (18) by numerical integration, and of perturbed output variables y_p from Eq. (19), is straightforward. However, computation of the corrected state variables in Eq. (20)

additionally requires perturbed gain matrix K_p, which is different for each parameter perturbation. It is given by

$$K_p = P_p C_p^T R^{-1} \tag{21}$$

where the covariance matrix of the state error P_p corresponding to the perturbed parameter is obtained by solving the Riccati equation (11) with system matrices computed for the corresponding perturbations. The elements of linearized perturbed system matrices A_p and C_p corresponding to perturbation $\delta\beta$ are once again approximated by central-difference formulas given in Eqs. (15) and (16), evaluated at $\beta+\delta\beta$. For the other elements of Θ, i.e., for λ, x_0, Δz, and Δu, equations similar to Eqs. (18–21) incorporating corresponding perturbations are used for propagation.

The extension of finite-difference approximation to filter algorithm thus involves not only the numerical integration of the perturbed state equations, but also additionally the computation of the perturbed gain matrices for each element of the unknown parameter vector Θ. The perturbed output variables y_p computed from Eq. (19) using perturbed states x_p and perturbed gain K_p will automatically account for the respective gradients. Thus, in comparison to the previous formulation for linear systems,[5,13] solution to a set of Lyapunov equations for a gradient of P and, from that, computation of gradient of K, has been replaced in the present approach by solutions to the perturbed Riccati equations. Any increase in the computational load due to this change will not be significantly high.

Although the system model in Eqs. (1–3) has been postulated in terms of the process and measurement noise matrices F and G, independently, the cost function to be minimized, Eq. (5), is defined in terms of R, the covariance of residuals. The two-step optimization procedure yields iteratively the parameter vector Θ and the covariance matrix R. This has been done with a view to eliminate the convergence difficulties,[5] which are generally encountered in the iterative estimation of the measurement noise covariance matrix GG^T using the Gauss-Newton method. Hence, in the present case, GG^T can only be indirectly obtained from the relation

$$R = GG^T + CPC^T \tag{22}$$

For physically meaningful results, it is necessary to insure that the measurement noise covariance matrix GG^T, as will be obtained indirectly from the preceding equation, is positive semidefinite. It has been shown in Ref. 5 that for linear systems the preceding conditions are well approximated by constraining the diagonal elements of the matrix KC to be less than unity. Thus, minimization of the nonlinear cost function, subject to these inequality constraints, leads to a nonlinear programming problem. A quadratic programming method can be used to solve this optimization problem.

In the present case, a similar approach has been adopted with a further simplification that the elements of matrix KC are constrained to be less than unity, where the observation matrix C obtained by first-order system approximation is used. The computational details of the constrained optimization are similar to those found in Refs. 5 and 13 and, hence, are not discussed further in this paper.

In the derivation of the ML estimation algorithm, it has been hitherto assumed that the covariance matrix of residuals R is known. In practice, however, this is hardly the case. Hence, it is generally required to estimate the matrix R also, like other unknown system parameters. The ML estimate of R can be obtained by equating to zero the gradient of the cost function with respect to R. Since the residuals are functions of gain matrix K, the exact equation for R is complex and computationally tedious. However, the asymptotic approximation to R can be obtained as[5]

$$R = \frac{1}{N}\sum_{k=1}^{N} [z(t_k)-y(t_k)][z(t_k)-y(t_k)]^T \tag{23}$$

The two steps to compute $\Delta\Theta$ and R, Eqs. (14) and (23), are carried out independently. Hence, they do not account for the influence of each on estimates of the other. This often yields strongly correlated estimates of matrices F and R, which affects the convergence. To account for this correlation effect, the following approximate procedure, heuristically suggested in Ref. 5, is used here to compensate for the F matrix whenever the covariance matrix of residuals R is revised.

$$F_{ij}^{\text{new}} = F_{ij}^{\text{old}}\left[\left(\sum_k C_{ki}^2\, r_k^{\text{old}}\sqrt{r_k^{\text{old}}/r_k^{\text{new}}}\right)\bigg/\left(\sum_k C_{ki}^2\, r_k^{\text{old}}\right)\right] \tag{24}$$

where r_k is the kth diagonal element of inverse of matrix R, superscripts "old" and "new" denote the previous and revised estimates, respectively.

From the preceding development, it is now obvious that the adapted formulation computes explicitly the covariance matrix of the residuals (innovation formulation) as a first step. In the second step, instead of the filter gain matrix K, the state noise matrix F is estimated along with the other parameters using the Gauss-Newton method (natural formulation). The main difficulties in estimating the gain matrix, normally encountered in the innovation formulation, are thus avoided. Although F is an $(n \times n)$ matrix [Eq. (1)], it is common practice in estimation to treat it as a diagonal matrix. This simplification not only helps to reduce the computational burden, but also to avoid any identification problems.

Examples

In this section, the extended formulation for nonlinear systems, discussed previously, has been tested on a number of flight mechanical examples. The following three examples help to validate the formulation and also aid in its evaluation.

Lateral Directional Motion—Simulation with Process Noise

To evaluate the performance of the estimation algorithm on data containing an appreciable level of turbulence, typical aircraft responses pertaining to the lateral-directional motion are generated through simulation. The nominal values of the aerodynamic derivatives used in simulation correspond to those obtained by parameter estimation from flight data recorded during the flight tests in a steady atmosphere with the De Havilland DHC-2 research aircraft.[19] Standard equations of aircraft motion, incorporating additional state and measurement noise, are used to generate the data. To provide realistic control inputs, signals corresponding to the rudder and aileron excitations actually applied in a particular flight test are used.

For the purpose of simulation, independent process and measurement noise vectors are generated using a pseudorandom noise generator. The state noise matrix F is assumed to be a diagonal matrix. A total of 16 s of data with a sampling time of 50 ms is generated. These typical noisy responses are used as measured data for the purpose of parameter estimation.

The following linear model pertaining to the lateral-directional motion of an aircraft is used to estimate the dimensional derivatives.

State equations:

$$\dot{p} = L_p p + L_r r + L_{\delta_a}\delta_a + L_{\delta_r}\delta_r + L_v v + b_{x_p} \tag{25a}$$

$$\dot{r} = N_p p + N_r r + N_{\delta_a}\delta_a + N_{\delta_r}\delta_r + N_v v + b_{x_r} \tag{25b}$$

Observation equations:

$$\dot{p}_m = L_p p + L_r r + L_{\delta_a}\delta_a + L_{\delta_r}\delta_r + L_v v + b_{y_{\dot{p}}} \tag{26a}$$

$$\dot{r}_m = N_p p + N_r r + N_{\delta_a}\delta_a + N_{\delta_r}\delta_r + N_v v + b_{y_{\dot{r}}} \tag{26b}$$

$$a_{ym} = Y_p p + Y_r r + Y_{\delta_a}\delta_a + Y_{\delta_r}\delta_r + Y_v v + b_{a_y} \tag{26c}$$

$$p_m = p + b_{y_p} \tag{26d}$$

$$r_m = r + b_{y_r} \tag{26e}$$

Table 1 Estimation of lateral-directional derivatives

		Estimates (standard deviations in %) obtained by accounting for both process and measurement noise	
Parameter	Nominal value	Algorithm for linear systems	Algorithm for nonlinear systems
L_p	−5.820	−5.719 (6.5)	−5.724 (6.6)
L_r	1.782	1.720 (8.9)	1.722 (9.1)
N_p	−0.665	−0.621 (9.3)	−0.621 (9.3)
N_r	−0.712	−0.722 (3.4)	−0.722 (3.4)
L_{δ_a}	−16.434	−14.925 (11.)	−14.928 (11.)
L_{δ_r}	0.434	0.200 (21.)	0.199 (21.)
L_v	−0.097	−0.088 (12.)	−0.088 (12.)
N_{δ_a}	−0.428	−0.426 (58.)	−0.427 (59.)
N_{δ_r}	−2.824	−2.828 (2.3)	−2.828 (2.4)
N_v	0.008	0.009 (18.)	0.009 (19.)
Y_p	−0.278	−0.297 (27.)	−0.297 (27.)
Y_r	1.410	1.415 (2.5)	1.415 (2.5)
Y_{δ_a}	−0.447	−0.514 (72.)	−0.514 (73.)
Y_{δ_r}	2.657	2.688 (3.7)	2.688 (3.7)
Y_v	−0.180	−0.180 (1.5)	−0.180 (1.4)
F_{11}	0.200	0.167 (4.5)	0.167 (4.2)
F_{22}	0.200	0.124 (3.7)	0.124 (3.7)
$\lvert R \rvert$		7.31374	7.31368
Iterations		10	10
CPU time, s		24	49

System equations (25) and (26), incorporating two diagonal elements in the state noise matrix F, are used to estimate the dimensional derivatives ($L_p, N_p, Y_p, \ldots$) and bias terms b_x, b_y. The results of estimation accounting for both process and measurement noise using 1) the proposed formulation for nonlinear systems, and 2) the previous formulation for linear systems[5,13] are presented in Table 1. Nominal values of the derivatives are also provided in the same table.

Estimates of all the derivatives agree reasonably well with the nominal values. The match between the measured and estimated responses in Fig. 1 is very good. No numerical problems are encountered in the optimization. Convergence in both cases is achieved in 10 iterations.

In the case of linear systems, the finite-difference approximation of system matrices in Eqs. (10) and (12) is exact. In such a case, the filter implementation in Eqs. (6–8) reduces to the conventional Kalman filter and provides the optimal state estimates. As evident from Table 1, the results obtained for this linear example using the more flexible approach proposed in this paper match well with those obtained from a formulation for linear systems only. This helps to validate the algorithm computationally.

The algorithm using finite-difference approximations for gradients needs more computational time than the one using analytical differentiation. This is obvious because in the analytical approach the gradients are computed using the state transition matrix. Computation of the transition matrix does not involve extra costs, since it is already computed for prediction of the state variable. On the other hand, the finite-difference method requires numerical integration of the perturbed state equations, once for each unknown parameter. Further, approximations of the system matrices are also required. These overheads are unavoidable and have to be accepted in light of the more flexibility it provides in handling different model structures.

It has been observed that the output error method, which accounts for measurement noise only, is quite inadequate for the analysis of this data with an appreciable level of turbulence. Comparison of the results obtained by accounting for and by neglecting process noise for this example is found in Ref. 13.

For the preceding example and for the examples presented in the next section, a fourth-order Runge-Kutta method is used to integrate the system and the perturbed state equations.

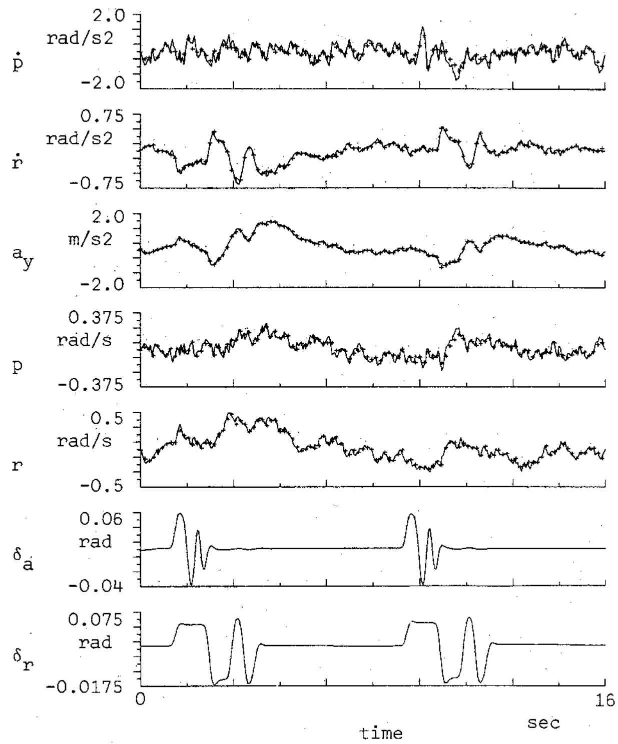


Fig. 1 Lateral directional motion variables [-------- measured (simulated); + + + + + estimated].

The response gradients are computed using a step size $\delta\Theta$ of $(10^{-6}\Theta)$. The first-order approximations of system matrices A and C in Eqs. (10) and (12) are obtained using a perturbation δx equal to $(10^{-4}x)$. All of the computations are carried out in double precision on a 32-bit computer.

Estimation of Longitudinal Derivatives

The second and third example, both presented in this section, pertain to estimation of the longitudinal aerodynamic derivatives of two research aircraft. Although a linear model can be formulated to extract such information, attention has been restricted in this paper to nonlinear models that are, in general, found to provide improved estimation results.[8,20,21]

It is possible to formulate the equations of aircraft motion to be used in estimation by using different coordinate systems.[3] The simplest of the various possible nonlinear models, containing only multiplicative and trigonometric nonlinearities, is obtained by defining the normalized aerodynamic forces X and Z and normalized pitching moment M as functions of variables in the body axes (u, w). Experience indicates that such a nonlinear model in terms of the dimensional derivatives ($X_u, Z_w, M_q, \ldots$) do not pose any specific difficulties in estimation. In many cases, however, it is of interest to obtain the nondimensional aircraft derivatives. Furthermore, the use of state and observation equations formulated directly in terms of nondimensional coefficients generally provides improved estimation results.[8,20] Hence, two such models are considered here.

The equations of motion pertaining to the longitudinal mode are given by the following.

State equations:

$$\dot{u} = -qw - g\,\sin\theta + \frac{F_e}{m}\cos\sigma_T + \frac{\rho V^2 S}{2m} C_X, \qquad u(0) = u_0 \tag{27a}$$

$$\dot{w} = qu + g\,\cos\theta - \frac{F_e}{m}\sin\sigma_T + \frac{\rho V^2 S}{2m} C_Z, \qquad w(0) = w_0 \tag{27b}$$

$$\dot{\theta}=q, \qquad \theta(0)=\theta_0 \tag{27c}$$

$$\dot{q}=\frac{F_e}{I_y}(\ell_{tx}\sin\sigma_T+\ell_{tz}\cos\sigma_T)+\frac{\rho V^2 S\bar{c}}{2I_y}C_m, \qquad q(0)=q_0 \tag{27d}$$

Observation equations:

$$V_m=V \tag{28a}$$

$$\alpha_m=\alpha-\frac{x_\alpha q}{V} \tag{28b}$$

$$\theta_m=\theta \tag{28c}$$

$$q_m=q \tag{28d}$$

$$\dot{q}_m=\frac{\rho V^2 S\bar{c}}{2I_y}C_m+\frac{F_e}{I_y}(\ell_{tx}\sin\sigma_T+\ell_{tz}\cos\sigma_T) \tag{28e}$$

$$a_{x_m}=\frac{\rho V^2 S}{2m}C_X+\frac{F_e}{m}\cos\sigma_T \tag{28f}$$

$$a_{z_m}=\frac{\rho V^2 S}{2m}C_Z-\frac{F_e}{m}\sin\sigma_T \tag{28g}$$

with

$$V=\sqrt{u^2+w^2} \tag{29a}$$

$$\alpha=\tan^{-1}\left(\frac{w}{u}\right) \tag{29b}$$

Flight tests were carried out with research aircraft HFB-320.[8,20] The short period and phugoid modes are excited with an elevator input consisting of a multistep signal followed by a longer-duration pulse. The input amplitudes are limited to result in a typical α variation of less than 10 deg. For these typical flight experiments, the normalized aircraft body-axis force coefficients C_X and C_Z and normalized pitching moment coefficient C_m are defined as

$$C_X=C_{X_0}+C_{X_u}\frac{u}{V_0}+C_{X_w}\frac{w}{V_0} \tag{30a}$$

$$C_Z=C_{Z_0}+C_{Z_u}\frac{u}{V_0}+C_{Z_w}\frac{w}{V_0} \tag{30b}$$

$$C_m=C_{m_0}+C_{m_u}\frac{u}{V_0}+C_{m_w}\frac{w}{V_0}+C_{m_q}\frac{q\bar{c}}{2V_0}+C_{m_{\delta e}}\delta_e \tag{30c}$$

The model postulated in Eqs. (27–29) also contains, apart from the common trigonometric and multiplicative nonlinearities, also those introduced by the variable dynamic pressure $\bar{q}=(1/2\ \rho V^2)$, which multiplies each aerodynamic derivative [Eq. (30)]. In addition, the use of directly measured variables V and α introduce further nonlinearities. However, since the algorithm is designed to handle nonlinear systems, no specific user manipulations are necessary.

A record length of 60 s with a sampling time of 0.1 s is used to estimate the nondimensional aircraft derivatives $C_{X_{()}}$, $C_{Z_{()}}$, and $C_{m_{()}}$. In addition, the unknown initial conditions u_0, w_0, θ_0, q_0 are also to be simultaneously estimated. Further, in the case of estimation accounting for process noise, diagonal elements of the process noise matrix, four in the present case, are also unkown.

The results of parameter estimation accounting for measurement noise only and for both process and measurement noise are provided in Table 2 and in Figs. 2a and 2b. The convergence in these two cases is achieved in 10 and 8 iterations. The agreement between the measured and the estimated responses in Fig. 2b is very good. The comparison of Figs. 2a and 2b clearly brings out the improvements obtained by accounting for the process noise. These qualitative improvements observed in Fig. 2 are corroborated quantitatively by the significant reduction in $|R|$ (Table 2) and by the fact that the residuals obtained by accounting for process noise were essentially white.

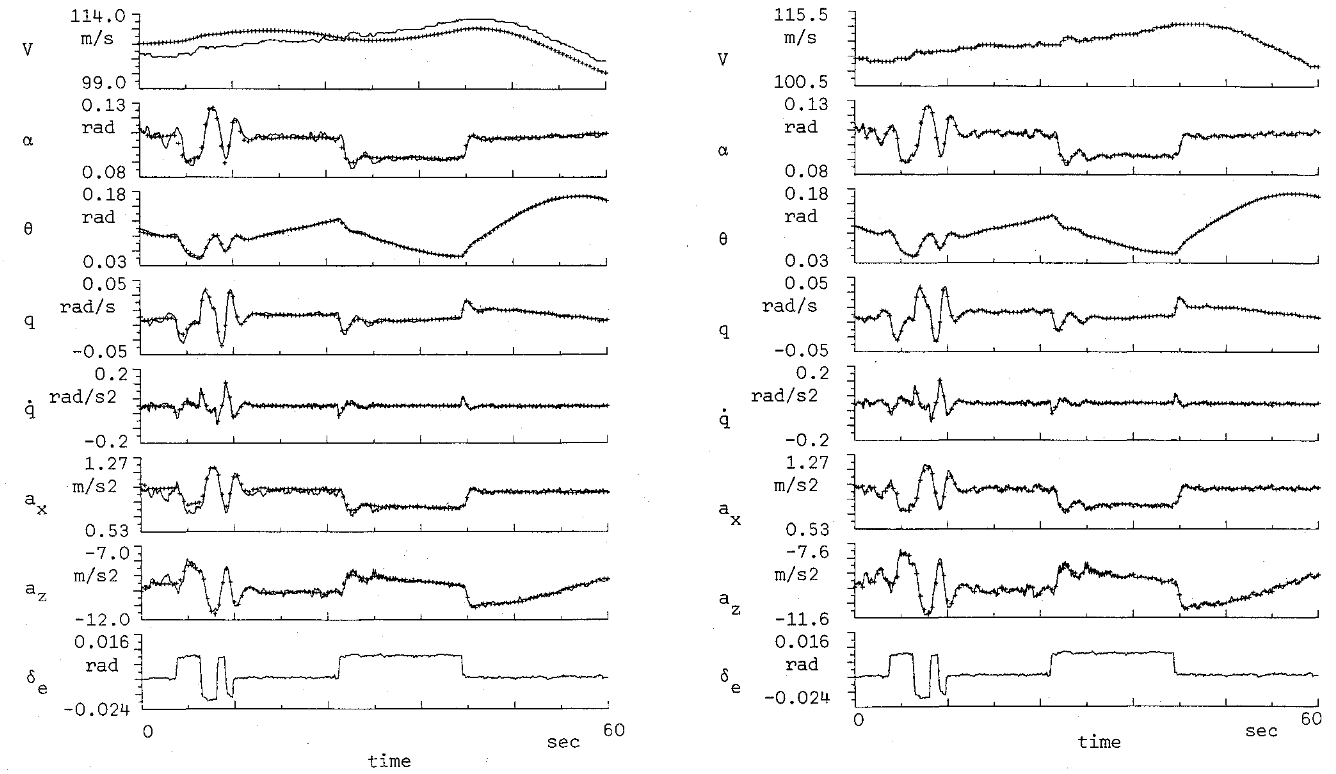
As a step further, the aerodynamic derivatives in wind-axis system (lift and drag derivatives) can now be obtained through standard axes transformations from the body-axis parameters estimated using Eqs. (27) and (30). It is also possible to estimate directly the lift and drag derivatives by incorporating such transformations into the postulated system model. Such a model, in terms of nondimensional lift, drag, and pitching moment coefficients (C_L, C_D, C_m) as functions of variables in the wind axis (V, α, etc.), results in system equations now containing nonlinearities, not only due to the dynamic pressure $\bar{q}$, but also those introduced by the axis transformations.[3,8] Further, inversions of the state variable V are required in state equations. Thus, model formulation in wind axis to extract aerodynamic derivatives leads to additional nonlinearities in the postulated model.

Although the details of parameter estimation using such a model in wind axis are not further discussed in this paper, it is worthwhile to mention here that this model containing more nonlinearities compared with that of Eqs. (27–30) posed no numerical difficulties in either the state estimation or optimization. The time plots of the measured and estimated responses obtained in this case were essentially the same as those in Fig. 2.

In the preceding model [Eq. (30)], the Taylor series expansion, leading to a model that is linear-in-parameters, has been used to estimate the aerodynamic derivatives. This conventional approach, often referred to as a derivative model, although adequate in many applications yields only an overall aircraft characteristic. Physically more realistic representation of aerodynamics, for example, pertaining to the longitudinal motion, however, involves individual modeling of wing/body and of horizontal tail surface. The use of such a "two-point aerodynamic model" in parameter estimation has been recently demonstrated in Ref. 22. Such a model, which is nonlinear not only in the equations of motion but also in aerodynamic parameters, is considered here to evaluate the performance of the algorithm proposed in this paper.

Table 2 Estimated nondimensional longitudinal derivatives, flight test data, HFB-320 aircraft

Parameter	Estimates (standard deviations in %) obtained by accounting for: Measurement noise only	Both process and measurement noise
C_{X_u}	0.0208 (23.)	0.0141 (15.)
C_{X_w}	0.5587 (1.8)	0.6179 (1.1)
C_{Z_u}	0.2527 (9.7)	0.2847 (5.5)
C_{Z_w}	−4.2332 (1.0)	−4.2039 (1.1)
C_{m_u}	0.0662 (6.6)	0.0956 (3.5)
C_{m_w}	−0.9780 (0.8)	−0.9274 (1.1)
C_{m_q}	−21.9740 (2.0)	−32.7070 (2.4)
C_{X_0}	−0.1160 (4.4)	−0.1162 (2.1)
C_{Z_0}	−0.3228 (7.7)	−0.3589 (4.9)
$C_{m_{\delta_e}}$	−1.4301 (0.9)	−1.5054 (1.3)
C_{m_0}	0.0528 (8.6)	0.0166 (21.)
F_{11}	—	0.3701 (4.7)
F_{22}	—	0.2763 (3.0)
F_{33}	—	0.0005 (10.)
F_{44}	—	0.0017 (5.9)
$\|R\|$	3.2849D-5	9.9858D-12
Iterations	10	8



a) Estimation accounting for measurement noise only

b) Estimation accounting for both process and measurement noise

Fig. 2 Curve fits from parameter estimation. Flight test data—HFB aircraft (------- measured; + + + + estimated).

The lift, drag, and pitching moment coefficients are modeled as[22]

$$C_L = C_{L_0} + C_{L_{\alpha WB}}\alpha + \frac{S_H}{S}\left\{C_{L_{\alpha H}}\left(\alpha - \frac{\partial \epsilon_H}{\partial \alpha}\alpha + i_H + \tan^{-1}\frac{qr_H}{V}\right) + K_{\delta_e}\delta_e + K_F F_e\right\} \tag{31a}$$

$$C_D = C_{D_0} + K_D C_L^2 \tag{31b}$$

$$C_m = C_{m_0} - \frac{r_H^*}{\bar{c}}\frac{S_H}{S}\left\{C_{L_{\alpha H}}\left(\alpha - \frac{\partial \epsilon_H}{\partial \alpha}\alpha + i_H + \tan^{-1}\frac{qr_H}{V}\right) + K_{\delta_e}\delta_e + K_F F_e\right\} + C_{m_{qWB}}\frac{q\bar{c}}{V} \tag{31c}$$

where the subscript "WB" refers to the wing-body combination and "H" to the horizontal tail surface. In addition to the already defined variables, ϵ_H denotes the downwash angle, S and S_H are the surface area of the wing and horizontal tail, respectively, r_H and r_H^* are the lever arms, respectively, from the center of gravity and the neutral point of the wing to the neutral point of the horizontal tail surface, and i_H is the tail-plane trim angle.

The body-axis force coefficients C_X and C_Z required in the postulated model [Eqs. (27-29)] are obtained from the axis transformations:

$$C_X = C_L \sin\alpha - C_D \cos\alpha \tag{32a}$$

$$C_Z = -C_L \cos\alpha - C_D \sin\alpha \tag{32b}$$

A typical time segment recorded during the flight tests with the research aircraft ATTAS is analyzed here using the preceding postulated model. The details of the flight test are found in Ref. 22. The data analyzed corresponds to a segment in which the DLC flaps are held fixed, and the aircraft motion is excited through variations in the thrust F_e, trim angle i_H, and elevator position δ_e. Since the pitch acceleration has not been measured directly, the vertical acceleration measured at the cockpit a_{zck}, has been used in the parameter estimation instead of $\dot{q}_m$ [Eq. (28)].

$$a_{zck_m} = \frac{\rho V^2 S}{2m} C_Z - \frac{F_e}{m}\sin\sigma_T - x_{ck}\dot{q} \tag{33}$$

The system model, in this case consisting of Eqs. (27-29) together with Eqs. (31-33), is nonlinear in the state and input variables and in the parameters. It is fairly straightforward to see that all the state equations and the observation equations for accelerations contain quadratic nonlinearity in V. The lift coefficient C_L, although linear in motion variables except for V, is nonlinear in parameters $C_{L_{\alpha H}} \cdot \partial\epsilon_H/\partial\alpha$. Further, the drag coefficient C_D depends quadratically on the lift coefficients and thus is a nonlinear function. The transformations of (C_L, C_D) in Eqs. (32) together with the small angle approximation suggest that the body-axis force coefficients C_X and C_Z are approximate functions of $(\alpha^2, \alpha\delta_e, \ldots)$ and $(\alpha^3, \alpha\delta_e^2, \ldots)$, respectively.

A 60-s long record with a sampling time of 80 ms has been used to estimate the following unknown aerodynamic parameters:

$$\Theta^T = \{C_{L_{\alpha WB}}, C_{L_{\alpha H}}, \frac{\partial \epsilon_H}{\partial \alpha}, K_D, K_{\delta_e}, C_{m_{qWB}}, C_{L_0}, C_{D_0}, C_{m_0}\}^T \tag{34}$$

Apart from the preceding parameters, the four unknown initial conditions $(u_0, w_0, \theta_0, q_0)$ and the measurement biases in the three accelerometers (a_x, a_z, a_{zck}) are to be estimated. The thrust parameter K_F has been kept constant. The variations in the altitude during the tests were only minor and, hence, a constant value for the density of air, ρ, has been used in the present analysis.

In this case of parameter estimation accounting for process noise, four parameters corresponding to the diagonal elements of process noise matrix F are additionally estimated. In the absence of a priori knowledge, very small values (0.002) are taken as the initial starting values of these diagonal elements. Further, the use of a process noise option requires the covariance matrix of residuals to be specified. This has been generated assuming the filter-gain matrix to be zero in the first pass. This is equivalent to treating the complete noise as measurement noise in the first iteration.

The parameter estimation results obtained by accounting for measurement noise only (output error method) and by accounting for both process and measurement noise are presented in Table 3 and in Figs. 3a and 3b. The time plots provided include the recomputation of angles from radians to degrees and of acceleration into g. Further, the convention of normal acceleration a_n has been included for the measured vertical acceleration.

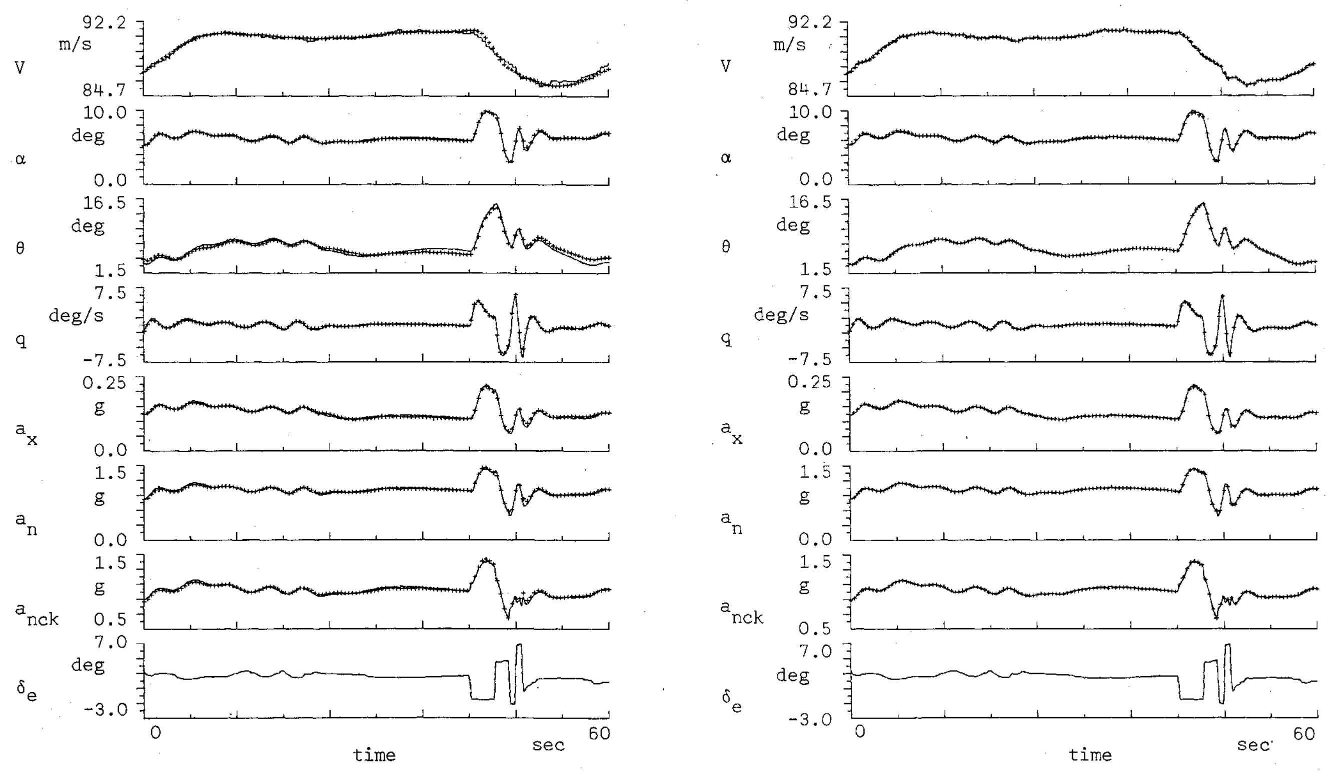
Table 3 Longitudinal parameters from flight test data—ATTAS aircraft, two-point model structure

Parameter	Estimates (standard deviations in %) obtained by accounting for: Measurement noise only	Both process and measurement noise
$C_{L_{\alpha WB}}$	5.0627 (0.5)	5.4559 (0.4)
$C_{L_{\alpha H}}$	2.4932 (1.1)	3.4738 (1.9)
$\partial\epsilon_H/\partial\alpha$	0.5378 (1.0)	0.6486 (1.1)
K_{δ_e}	1.5666 (0.5)	1.5665 (0.6)
K_D	0.0520 (1.3)	0.0401 (1.1)
$C_{m_{qWB}}$	−2.0517 (4.3)	−0.0015 (**.)
C_{L_0}	0.2584 (1.1)	0.2229 (1.3)
C_{D_0}	0.0650 (0.9)	0.0641 (1.9)
C_{m_0}	0.0579 (1.3)	0.0357 (5.7)
u_0	86.7860 (0.03)	86.7264 (0.10)
w_0	7.7522 (0.61)	8.0960 (0.62)
θ_0	0.0756 (0.96)	0.0606 (0.36)
q_0	−0.0221 (4.27)	−0.0203 (4.76)
$\lvert R \rvert$	1.95970D+3	8.43441D-6
Iterations	7	6

No numerical difficulties either in the filter algorithm or in the estimation of parameters and of noise covariances were encountered. The comparison of time histories of the measured and estimated responses in Fig. 3, and the comparison of the determinant of R in Table 3, clearly indicate that appreciable improvements are obtained by accounting for process noise, even for this flight test data in seemingly steady atmosphere. This is because the process noise formulation not only enables one to account for the atmospheric turbulence, but also helps to better account for the modeling errors, compared to the output error method.[5] Further discussion of the flight mechanical aspects of the preceding results is not pursued in this paper, since the results presented adequately demonstrate the applicability of the proposed estimation algorithm to nonlinear systems with process noise, even for fairly complex system models as the aforementioned one.

The performance of the proposed extension using a filter based on a first-order system approximation of nonlinear model postulates has been surprisingly good. This is particularly noteworthy in the case of the last two nonlinear models presented. This is attributed to the fact that the data analyzed was recorded during the flight tests carried out specifically for the purpose of parameter estimation. As such the deviations from the trim conditions are not large, for example, variations in α less than 10 deg. In such a case, i.e., for moderately nonlinear systems, the approximate filter appears to be quite adequate, and this leads to a very flexible and computationally attractive estimation algorithm for nonlinear systems with process noise.

In the cases where the deviations from the trim conditions are likely to be large, such as large-amplitude maneuvers, the use of an approximate filter with constant gain may require some considerations. If a filter divergence is encountered in



a) Estimation accounting for measurement noise only

b) Estimation accounting for both process and measure noise

Fig. 3 Curve fits from parameter estimation. Flight test data—ATTAS aircraft (------- measured; + + + + estimated).

such cases, a time-varying filter may have to be incorporated. This will, however, result in a considerable increase in the computational load. Investigations of such problems are currently being pursued.

Conclusions

The algorithm for parameter estimation in dynamic systems with both process and measurement noise, hitherto applied only to linear systems, has been suitably extended in this paper to nonlinear systems. This extension can be conveniently used for linear and moderately nonlinear systems. It incorporates a constant-gain filter based on a first-order system approximation about the initial conditions. The point of linearization, i.e., the initial conditions, are updated iteratively, along with the other unknown system parameters, in a parameter update loop using the Gauss-Newton method. For linear systems, the approximate filter reduces to the Kalman filter, which is an optimal state estimator. The preceding extended algorithm has been used to estimate flight mechanical parameters from simulated aircraft responses and from flight test data using both linear and nonlinear system models.

As a typical example, the estimation of the longitudinal aerodynamic parameters from flight test data using various model postulates with different degrees of nonlinearities in the state and control variables and in the parameters has been investigated in some depth. The performance of the estimation algorithm incorporating a nonlinear filter with constant gain has been found to be very good in all of the examples reported.

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